An Improved Compact 2-D Finite-Difference Frequency-Domain Method for Guided Wave Structures

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***Abstract*—An improved compact two-dimensional (2-D) finite-difference frequency-domain method is presented to determine the dispersion characteristics of guided wave structures. Eigenvalue equations that contain only two transverse electric field components are derived from Maxwell's differential equations. Compared to the traditional 2-D FDFD containing four or six field components, both the order and number of nonzero elements of the coefficient matrix are reduced simultaneously. The method is verified by two application examples.**

***Index Terms*—Eigen matrix, finite-difference frequency domain, order reduction.**

I. Introduction

Many full-wave electromagnetic analysis methods have been developed to determine the electrical characteristics of guided wave structures. Some typical methods include finite difference time domain (FDTD) [1], method of moment (MoM) [2] and finite element method (FEM) [3] etc. Because of its versatility in handling arbitrary structures, FDTD has attracted much attention. However, 3-dimensional FDTD generally consumes much memory and CPU time. To overcome this problem, a two-dimensional (2-D) FDTD approach has been introduced by S. Xiao [4], [5]. In the 2-D FDTD, the guided wave structures are excited with a time-domain impulse at a given propagation constant β, and the eigen frequencies of interest are then extracted through Fourier transform of the time-domain responses. The 2-D mesh consisting of six field quantities reduces the computer memory requirement dramatically. However, the difficulty is that the grid size must be decided at the beginning, yet the choice of the size is dependent on the frequencies. Improper selection of the mesh grid size would cause instability problems [6].

Recently, a compact 2-D finite difference frequency domain (FDFD) has been proposed [7], in which the propagation constant β is determined for a given frequency. To improve the efficiency of 2-D FDFD, the number of electromagnetic field components that are evolved in the eigenvalue equation is reduced from six to four in [8]. In this letter, the efficiency of 2-D FDFD is further improved by reducing both the order and number of nonzero elements of the coefficient matrix. For a given frequency, the eigenvalue equation consists of only two transverse electric field components by eliminating the magnetic fields based on Maxwell's equations. Compared to the existing 2-D FDFD that contains four [8] or six [7] field components, the order and number of nonzero elements of the coefficient matrix are reduced simultaneously. The reliability and efficiency of the improved FDFD are verified by two application examples.

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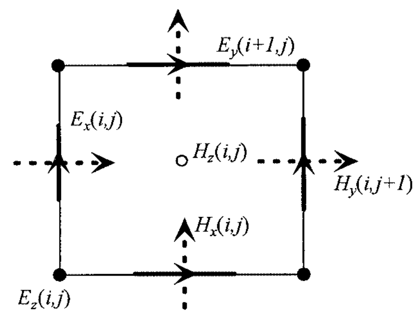

Fig. 1. The compact 2-D mesh.

II. Formulation

The differential Maxwell equations in the frequency domain are

$$\nabla \times \mathbf{E} = -j\omega\boldsymbol{\mu}\cdot\mathbf{H} \tag{1-a}$$

$$\nabla \times \mathbf{H} = j\omega\boldsymbol{\varepsilon}\cdot\mathbf{E} \tag{1-b}$$

$$\nabla\,\mathbf{D} = 0 \tag{1-c}$$

$$\nabla\,\mathbf{B} = 0 \tag{1-d}$$

Assuming that the guided wave structure is uniform along z-axis and the wave propagates in +z direction. By replacing the propagation direction derivative with $-\mathrm{j}\beta$ (β is the propagation constant) and using the compact 2D mesh in Fig. 1 as that in [7], three scalar equations of (1-a) and (1-b) are discretized in Cartesian coordinates by central difference as (only diagonal dielectric constant tensor $\bar{\bar{\varepsilon}}_r$ is considered for simplicity):

$$j\omega\varepsilon_{xx}E_x(i,j) = \frac{H_z(i,j)-H_z(i,j-1)}{\Delta y} + j\beta H_y(i,j) \tag{2-a}$$

$$j\omega\mu H_z(i,j) = \frac{E_x(i,j+1)-E_x(i,j)}{\Delta y} - \frac{E_y(i+1,j)-E_y(i,j)}{\Delta x} \tag{2-b}$$

$$j\omega\mu H_y(i,j) = \frac{E_z(i+1,j)-E_z(i,j)}{\Delta x} + j\beta E_x(i,j) \tag{2-c}$$

in which ε_{xx} is the dielectric permittivity in the x direction, Δx, Δy are grid sizes in the x and y directions, respectively.

Substituting (2-b) and (2-c) into (2-a) yields

$$a_1E_y(i+1,j)+a_2E_x(i,j+1)+a_3E_y(i,j)+ a_4E_z(i+1,j)+a_5E_z(i,j)+a_6E_y(i+1,j-1)+ a_7E_y(i,j-1)+a_8E_x(i,j-1)+a_9E_x(i,j)=0 \quad (3)$$

Equation (3) consists of only E field components. In a similar way, (1-c) is discretized as

$$\frac{D_x(i,j)-D_x(i+1,j)}{\Delta x}+\frac{D_y(i+1,j-1)-D_y(i+1,j)}{\Delta y} +j\beta D_z(i+1,j)=0 \quad (4)$$

where $D_l(i,j,k)=\varepsilon_{ll}E_l(i,j,k)$, $l=x,y,z$. ε_{ll} is the dielectric permittivity of different direction that can be dealt with as in FDTD at the boundary of two dielectric mediums.

Now substituting (4) into (3), we get an expression describing the relationship between neighbor transverse electric fields:

$$A_{x,1}E_x(i+1,j)+A_{x,2}E_x(i-1,j)+ A_{x,3}E_x(i,j-1)+A_{x,4}E_x(i,j+1)+ A_{x,5}E_y(i+1,j-1)+A_{x,6}E_y(i+1,j)+ A_{x,7}E_y(i,j-1)+A_{x,8}E_y(i,j)+ [A_{x,9}-\beta^2]E_x(i,j)=0 \quad (5)$$

where $A_{x,i}$ $(i=1\sim 9)$ are coefficients, for example,

$$A_{x,1}=-\frac{2(\varepsilon_{xx}(i,j)+\varepsilon_{xx}(i,j+1))}{\varepsilon_{zz}(i,j)+\varepsilon_{zz}(i,j-1)+\varepsilon_{zz}(i+1,j)+\varepsilon_{zz}(i+1,j-1)} \times\left(\frac{1}{\Delta x}\right)^2 \quad (6)$$

Considering that the longitudinal field component E_z has been eliminated in (5), the boundary condition $E_z=0$ on the surface of ideal conductors can be implanted in (3) and (4), and (5) is then modified accordingly.

Another expression about the relationship between neighbor transverse electric fields can be derived similarly by discretizing another three scalar equations of (1-a) and (1-b) and using (4):

$$A_{y,1}E_y(i,j+1)+A_{y,2}E_x(i,j)+ A_{y,3}E_x(i-1,j)+A_{y,4}E_y(i+1,j)+ A_{y,5}E_y(i-1,j)+A_{y,6}E_x(i,j+1)+ A_{y,7}E_x(i-1,j+1)+A_{y,8}E_y(i,j-1)+ [A_{y,9}-\beta^2]E_y(i,j)=0 \quad (7)$$

in which $A_{y,i}(i=1\sim 9)$ are coefficients, too ...

After all boundary conditions (in this letter the boundary is supposed to be ideal metal for simplicity) are taken into account, combining (5) and (7) for all index i and j yields the following eigenvalue equation:

$$(\mathbf{A}-\lambda\cdot\mathbf{I})\,\vec{E}=0 \quad (8)$$

in which $\mathbf{A}$ is a highly sparse coefficient matrix whose elements are the coefficients in (5) and (7), $\lambda=\beta^2$, and $\vec{E}=\{E_x,E_y\}^T$ which contains only two transverse electric field components. When the eigenvalue λ is solved, the complex propagation constant β can be obtained consequently.

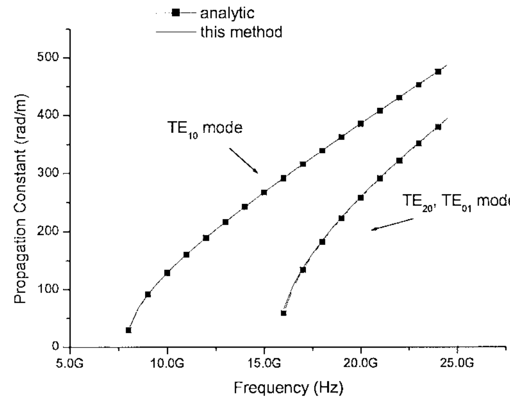

Fig. 2. Propagation constant of the rectangle waveguide.

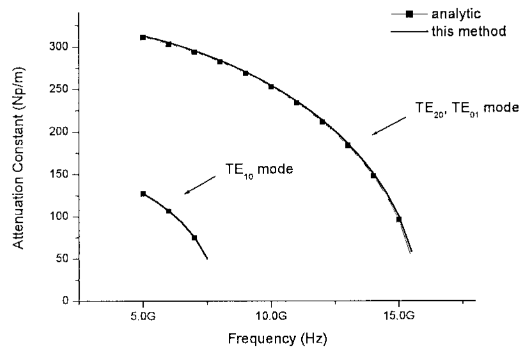

Fig. 3. Attenuation constant of the rectangle waveguide.

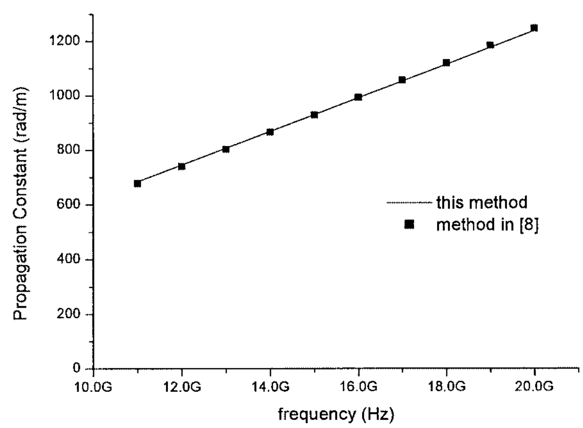

Fig. 4. Propagation constant of shielded microstrip on anisotropic substrate.

III. Numerical Results

In order to verify the validity and efficiency of the improved compact 2-D FDFD proposed in this letter, two application examples are given. The first is an empty rectangle waveguide with height $\mathrm{b}=9.525$ mm and width $\mathrm{a}=19.05$ mm. The analysis frequency ranges from the low end of the cutoff modes to the high end containing higher modes. The numerical results are compared with the analytic solution in Fig. 2 and Fig. 3. As shown in these two figures, good agreement is achieved.

The second example is a shielded microstrip line on electric anisotropic substrate. The trace is 1.5 mm wide and the thickness is negligible. The height of the substrate is 1.5 mm and the dielectric diagonal constant tensor is $\varepsilon_{yy}=11.6$ and $\varepsilon_{xx}=\varepsilon_{zz}=9.4$. The shielded metallic box is $9mm\times 6mm$. The numerical results are shown in Fig. 4 and compared with that in [8].

TABLE I
EFFICIENCY COMPARISON BETWEEN THIS METHOD AND THE METHOD IN [8] FOR THE SECOND EXAMPLE

	Mesh grid	Time (second)	Number of nonzero elements
This method	30×50	**22.312**	**15928**
	20×30	**7.191**	**6175**
Method in [8]	30×50	**130.708**	**39736**
	20×30	**28.882**	**15388**

Compared to the 2-D FDFD in [7] and [8], the dimension of the coefficient matrix $\mathbf{A}$ is reduced by two thirds and one half respectively, because only two field components are contained in the improved compact 2-D FDFD. Especially, the number of nonzero elements in the coefficient matrix in [8] is double of that in this method though the eigen matrix in [8] could be also composed of two field components by eliminating the H_x and H_y. The method in this letter cuts down both the order and number of nonzero elements of the coefficient matrix simultaneously. So the time and memory requirement are both reduced. Table I shows the efficiency comparison on Pentium IV 1.60 GHz PC.

IV. CONCLUSION

In this letter an improved compact 2-D FDFD has been proposed for calculating the dispersion characteristics of general guided wave structures. Compared to the existing 2-D FDFD that contains four or six field components, both the order and number of nonzero elements of the coefficient matrix are reduced obviously, which means that much computer memory and CPU time can be saved.

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